Excitation by scattering/total field decomposition and UPML in the geometric formulation

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The paper presents a general technique to apply excitations in the framework of discrete geometric numerical methods using dual grids, like DGA and FIT.

***Index Terms*—Discrete Geometric Approach, Waveguide, Modal Excitation, Port boundary condition.**

I. Introduction

In our previous works [1], [2] it was shown how a *plane wave* excitation can be integrated in the framework of the Discrete Geometric Approach (DGA). That purpose was attained by using an *impedance boundary condition* which allowed to simulate the plane wave. Such excitation, however, is not adequate for use with waveguides since, in this case, it can excite only the fundamental mode. Moreover that formulation does not allow the presence of objects producing scatterings in the surroundings of the boundary. Therefore, the aim of this paper is to present a novel and more general technique to apply excitations, based on a *scattered/total field decomposition*. That decomposition is obtained by splitting the simulation domain Ω in two subregions, the *scattering field* region Ω_s and the *total field* region Ω_t. Excitation is applied in the *interface* Σ between Ω_s and Ω_t by introducing an *additional dual boundary grid*. Therefore, the presented technique is applicable in the framework of numerical methods involving discretization with dual cell complexes, like DGA [3] and FIT [4].

In the following the discussion will be focused on a piece of rectangular waveguide but the technique can be applied to guides of arbitrary shape. The intuitive idea is to attach to the waveguide under study, which constitutes the total field region Ω_t, an additional region Ω_s, as detailed in (Fig. 1). Moreover, both Ω_s and Ω_t contain a *perfectly matched layer* subregion, to absorb the incident waves. In the following sections it will be described how the scattering field/total field subdivision is obtained and how the interface between Ω_s and Ω_t can be used to apply the desired mode. Finally, numerical results are described.

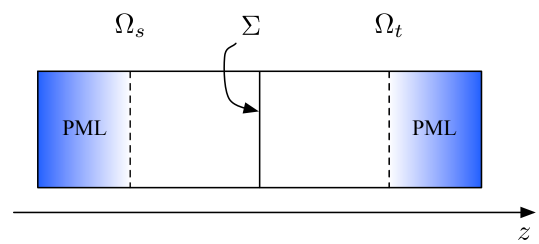

Fig. 1. The subregions of the problem: PMLs, scattered field region Ω_s and total field region Ω_t.

II. Electromagnetic wave propagation in frequency domain

The time-harmonic electromagnetic wave propagation in a region of space Ω is described in terms of the usual Maxwell differential formulation [5] as

$$\nabla \times (\boldsymbol{\nu} \nabla \times \boldsymbol{e}) - \omega^2 \boldsymbol{\epsilon} \boldsymbol{e} = \mathbf{0}, \tag{1}$$

where $\boldsymbol{\nu}$ and $\boldsymbol{\epsilon}$ are the material tensors, ω is the angular frequency and $\boldsymbol{e}$ is a complex-valued vector function describing the electric field. Numerical treatment of (1) requires the discretization of Ω, which is obtained by means of a *primal* tetrahedral grid $\mathcal{G}$ and a *dual* grid $\tilde{\mathcal{G}}$ induced by the barycentric subdivision of $\mathcal{G}$. Some electromagnetic quantities are associated with these interlocked grids, in particular

- Electromotive force U_k to edges $e_k \in \mathcal{G}$;
- Magnetic flux Φ_k to faces $f_k \in \mathcal{G}$;
- Magnetomotive force F_k to edges $\tilde{e}_k \in \tilde{\mathcal{G}}$;
- Electric flux Ψ_k to faces $\tilde{f}_k \in \tilde{\mathcal{G}}$.

Problem (1) is discretized as [2], [4]

$$\mathbf{C}^T \mathbf{M}_\nu \mathbf{C} \mathbf{U} - \omega^2 \mathbf{M}_\epsilon \mathbf{U} = \mathbf{0}, \tag{2}$$

where $\mathbf{C}$ is the face-edge incidence matrix, $\mathbf{M}_\nu$ and $\mathbf{M}_\epsilon$ are the constitutive matrices [6] and $\mathbf{U}$ is the array of the electromotive forces across the edges of the volume elements in which the entire domain Ω is discretized.

III. Scattered field/Total field formulation

In the Ω_t region the electromagnetic field is due to the excitation imposed on Σ and to the reflections that occur inside Ω_t (Fig. 2). On the other hand, the scattering field in the region Ω_s is only due to the reflections occurring in Ω_t and returning back to Ω_s. In both regions wave propagation occurs as prescribed by (1), but on Σ a transition from the total field to the scattered field happens and thus the tetrahedra in the Ω_s side touching Σ will require a special treatment,

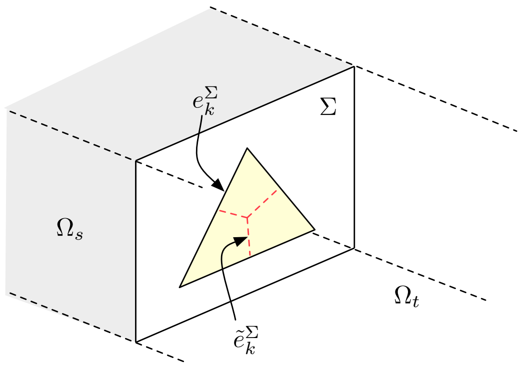

Fig. 2. Boundary primal edges e_k^Σ and dual edges $\tilde{e}_k^\Sigma$ introduced in the interface between total field and scattered field regions.

whose mathematical details will be given in the full paper. The intuitive idea is that on Σ, electromotive forces $U_{k,t}$ on $e_k^\Sigma \in \mathcal{G}$ can be decomposed in an excitation component $U_{k,i}$ and a reflected component $U_{k,s}$ such that $U_{k,t} = U_{k,i} + U_{k,s}$ holds. On the other hand, by introducing an *additional boundary dual grid* $\tilde{\mathcal{G}}^\Sigma$ on Σ, magnetomotive forces on $\tilde{e}_k^\Sigma \in \tilde{\mathcal{G}}^\Sigma$ are treated. On the edges of $\tilde{\mathcal{G}}^\Sigma$ we will show that the condition $F_{k,t} + F_{k,i} + F_{k,s} = 0$ must hold, where the three contributes to the circulation are due to the total field, to the excitation and to the scattered field respectively. Since the excitation is known, the condition on $\tilde{e}_k^\Sigma$ is rewritten as $F_{k,t} + F_{k,s} = -F_{k,i}$.

IV. Application of the excitation on the interface

At this point, to apply the desired excitation is a matter of setting the correct values for the electromotive and magnetomotive forces on both the primal and dual edges of Σ by fixing the values of $U_{k,i}$ and $F_{k,i}$. In the case of the rectangular waveguide of the example, these quantities are calculated by integrating on primal and dual mesh edges the field computed with closed-form equations for TE and TM modes. Conversely, in the case of an arbitrary shape of the waveguide, the values of $U_{k,i}$ and $F_{k,i}$ are computed by solving a 2D eigenvalue problem.

V. Numerical experiments

The presented technique was implemented in EMT, our DGA workbench code written in C++11. The simulations were performed on Mac OS X 10.9.5 running on a Core i7 3615QM with 16 GB of RAM, Clang/LLVM 3.5 compiler and MKL PARDISO solver. To test the technique a section of rectangular waveguide (Fig. 3) was simulated. The mesh included 178280 tetrahedra, which gave rise to a problem of 192242 unknowns. Problem assembly took 1.84 seconds while solver took 6.44 seconds. The waveguide dimensions were $a = 60mm$ (in x direction) and $b = 30mm$ (in y direction). PML regions length was $30mm$, scattered field region was $30mm$ and waveguide (total field) region length was $100mm$. Finally, the operating frequency was $f = 3.8GHz$.

As an additional test, power flowing in Σ was computed for different mesh sizes (Fig. 4).

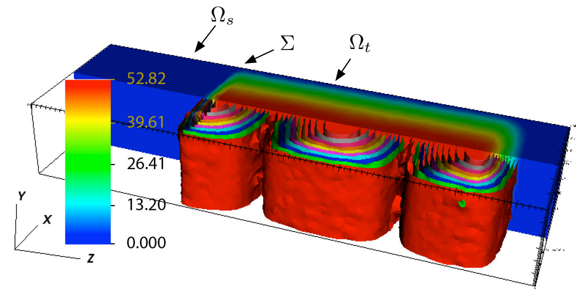

Fig. 3. Waveguide subject of the simulation, excited with TE_{10} mode on Σ. Scale shows the magnitude of the electric field in V/m (solid color region).

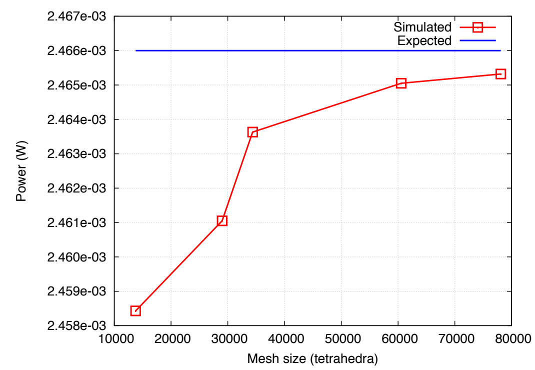

Fig. 4. Mesh size (number of tetrahedra) vs. power flowing in Ω_t towards positive z. Simulated power is compared with expected theoretical power.

VI. Conclusions

With this work a new tool was integrated in the DGA framework. It consists in a novel technique to apply excitations to the simulation domain Ω, in particular it allows to apply the correct excitations to waveguides. This was not possible with our previous work [2].

Acknowledgements

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References

[1] S. Chialina, M. Cicuttin, L. Codecasa, R. Specogna, F. Trevisan, *Port boundary condition for discrete electromagnetic problems in the frequency domain,* Proceedings of the CEFC2014 conference (ID 217). May 25-28, 2014, Annecy (France).

[2] S. Chialina, M. Cicuttin, L. Codecasa, R. Specogna, F. Trevisan, *Plane wave excitation for frequency domain electromagnetic problems by means of impedance boundary condition,* IEEE Transactions on Magnetics, In print (DOI: 10.1109/TMAG.2014.2358701).

[3] L. Codecasa, R. Specogna, F. Trevisan, *Discrete Geometric Formulation of Admittance Boundary Conditions for Frequency Domain Problems Over Tetrahedral Dual Grids,* IEEE Trans. Antennas Propagat., vol. 60, no. 8, pp. 3998–4002.

[4] T. Weiland, *Discrete electromagnetism with the finite integration technique,* Progress In Electromagnetics Research, 2001.

[5] Robert E. Collin, *Foundations for Microwave Engineering (Second edition),* McGraw-Hill International Editions, New York, 1992.

[6] L. Codecasa, R. Specogna, F. Trevisan, *Symmetric Positive-Definite Constitutive Matrices for Discrete Eddy-Current Problems,* IEEE Transactions on Magnetics, Vol. 43, No. 2, February 2007, pp. 510–515.

[7] A. Taflove, *Advances in Computational Electrodynamics,* Artech House Publishers, Boston, 1998